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## SECOND-QUANTIZED MIRROR SYMMETRY

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### Abstract

We propose and give strong evidence for a duality relating Type II theories on Calabi-Yau spaces and heterotic strings on  $K3 \times T^2$ , both of which have  $N = 2$  spacetime supersymmetry. Entries in the dictionary relating the dual theories are derived from an analysis of the soliton string worldsheet in the context of  $N = 2$  orbifolds of dual  $N = 4$  compactifications of Type II and heterotic strings. In particular we construct a pairing between Type II string theory on a self-mirror Calabi-Yau space  $X$  with  $h^{1,1} = h^{2,1} = 11$  and a  $(4,0)$  background of heterotic string theory on  $K3 \times T^2$ . Under the duality transformation the usual first-quantized mirror symmetry of  $X$  becomes a second-quantized mirror symmetry which determines nonperturbative quantum effects. This enables us to show that the quantum moduli space for this example agrees with the classical one. Mirror symmetry of  $X$  implies that the low-energy  $N = 2$  gauge theory is finite, even at enhanced symmetry points. This prediction is verified by direct computation on the heterotic side. Other branches of the moduli space, which are not finite  $N = 2$  theories, are connected to this one via black hole condensation.

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## 1. Introduction

It has been conjectured that many string theories in various dimensions have dual descriptions[1,2,3,4,5,6,7] which allow one to obtain information about the theory at strong coupling by performing weak coupling calculations in the dual theory. Currently the most compelling evidence exists for a string-string duality relating the IIA theory on  $K3$  to the heterotic string on  $T^4$ [8,3,6,9,10,11], which is a compactified version of string-fivebrane duality [2]. When these theories are further reduced to four dimensions by toroidal compactification, six-dimensional string-string duality implies  $S$ -duality of the resulting four-dimensional theories[12,4,6]. These four-dimensional theories have  $N = 4$  supersymmetry.  $S$ -duality in these theories is the natural generalization to supergravity and superstring theory of  $S$ -duality in  $N = 4$  Yang-Mills theory.

It has also been realized recently that duality plays a fundamental role in understanding the dynamics of gauge theories with  $N = 2$  [13] and  $N = 1$  [14] supersymmetry. It is natural to wonder whether duality in these theories might also have roots in duality in string theory. Since  $N = 2$  gauge theories are under the most precise control, this seems like a natural starting point for investigating this idea.

String theories in four dimension with  $N = 2$  supersymmetry arise either by compactification of Type II theories on Calabi-Yau spaces or by compactification of the heterotic string on  $K3 \times T^2$ . The question then is whether it is possible to find a duality relating Type II theory on a particular Calabi-Yau space to a heterotic string background on  $K3 \times T^2$ . In trying to make such a correspondence, a puzzle immediately arises. For  $N = 2$  compactification of a type IIA string,  $\chi = 2(N_v - N_h + 1)$ , where  $\chi$  is the Euler character of the Calabi-Yau space  $X$  and  $N_h = h^{2,1}(X) + 1$  ( $N_v = h^{1,1}(X)$ ) is the number of four dimensional hyper (vector) multiplets. On the other hand, it is easy to see that  $2(N_v - N_h + 1)$  is not constant over all branches of the moduli space of  $K3 \times T^2$  heterotic string compactifications: it can change at enhanced symmetry points<sup>5</sup>. Since  $\chi$  is constant over the moduli space of a single Calabi-Yau space, this cannot be represented by a type II string on a single Calabi-Yau space. Rather one requires a family of Calabi-Yau spaces. The type II description of changing  $2(N_v - N_h + 1)$  by passing through an enhanced symmetry point will involve jumping from one Calabi-Yau space to its neighbor via black hole condensation, as recently described in [15].

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<sup>5</sup> Of course this is just the classical picture. The connectivity of the various branches of the moduli space could differ quantum mechanically.

Given such a dual pair of string theories, the exact quantum moduli space can be determined from classical computations. The reason for this is that the dilaton is in a vector (hyper) multiplet on the heterotic (type IIA) side. Supersymmetry prevents couplings between neutral vector and hypermultiplets in the low-energy effective action [16]. Therefore the vector (hyper) multiplet geometry on the type IIA (heterotic) side can not depend on the string coupling, and is exact at tree level. The full quantum geometry is thus determined by a classical vector (hyper) multiplet computation in the type IIA (heterotic) representation of the theory. In particular spacetime instanton effects in the heterotic representation are worldsheet instantons in the dual type IIA representation, and so may be classically computed. This is second-quantized mirror symmetry. Second-quantized mirror symmetry can be used to sum up spacetime instanton corrections just as ordinary first-quantized mirror symmetry can be used to sum up worldsheet instanton corrections [17].

In this paper we will investigate in detail a specific branch of the  $N = 2$  moduli space. Although we will find a precise mapping and correspondence between dual theories in this example the general situation is not yet understood. We begin with the dual  $N = 4$  pair consisting of a IIA theory on  $K3 \times T^2$  and the heterotic theory on  $T^6$ . The IIA theory has a soliton string [9,10] (a compactification of the fivebrane of [18]) whose effective worldsheet theory is exactly that of a heterotic string on  $T^4$  [10]. Next we take a  $Z_2$  orbifold of the IIA theory, yielding an  $N = 2$  Calabi-Yau compactification on a manifold  $X$ . The corresponding orbifold on the heterotic side is then determined from the  $Z_2$  action on the soliton string. The dual pair so constructed indeed have the same values of  $N_h$  and  $N_v$ .

The fact that  $X$  is its own mirror is used to argue that there are no worldsheet or spacetime instanton corrections on this branch of the moduli space. This implies that the effective  $N = 2$  gauge theories at all enhanced symmetry points must lie in finite  $N = 2$  representations, a prediction which is verified by direct computation. For example we find the finite  $SU(2)$ ,  $N_f = 4$  theory, and our construction gives new insights into the beautiful results of [19]. For example, the triality discovered in [19] arises as the dual image of a  $T$ -duality transformation in the type II theory.

The suggestion of a duality relation between  $N = 2$  heterotic and type II compactifications was made previously in [20,21]. Perturbative aspects of  $N = 2$  heterotic compactifications were studied in [22]. Other examples of this duality including stringy analogs of Seiberg-Witten monopole points have been found recently in [23].

## 2. The Calabi-Yau Construction

We wish to construct a Calabi-Yau space as an orbifold of  $K3 \times T^2$ . As discussed in the introduction, we will obtain a specific test of string duality by constructing a Calabi-Yau space which is self-mirror. We will show that this is the case for a Calabi-Yau space  $X$  constructed as a freely acting orbifold of  $K3 \times T^2$ . Type II string propagation on  $X$  has only  $N = 2$  spacetime supersymmetry rather than  $N = 4$  spacetime supersymmetry. Such an orbifold can be constructed by utilizing a well-known freely acting involution  $\theta_1$  of certain  $K3$  surfaces. For  $K3$  surfaces admitting this involution the quotient is known as an Enriques surface [24]. Let  $z_3$  be a complex coordinate on  $T^2$ . We now mod out by  $\Theta = \theta_1 \theta_2$  where  $\theta_2$  acts as inversion on  $T^2$ ,  $\theta_2 z_3 \theta_2^{-1} = -z_3$ . The Hodge numbers of the Enriques surface are  $h_{11} = 10$ ,  $h_{02} = h_{20} = 0$ . In particular there is no holomorphic two-form. So the holomorphic two-form  $\Omega$  on  $K3$  is odd under  $\theta_1$  and thus the holomorphic three-form  $\Omega \wedge dz_3$  is preserved by  $\Theta$  and the quotient is a Calabi-Yau space with Euler number zero. It is important to note that the involution exists and this construction can be carried out on a large subspace of the  $K3$  moduli space. Generic points in this subspace do not contain any degenerations or enhanced symmetries.

Now by the Lefschetz fixed point theorem we have

$$\chi_{\theta_1} = 0 = 2 + Tr_2 \theta_1 \quad (2.1)$$

where  $\chi_{\theta_1}$  is the Euler character of the fixed point set of  $\theta_1$ , the 2 comes from  $H^0(K3)$  and  $H^4(K3)$  and  $Tr_2 \theta_1$  is the trace of the action of  $\theta_1$  on  $H^2(K3)$ . Thus  $\theta_1$  has eigenvalues  $[(-1)^{12}, (+1)^{10}]$  acting on  $H^2(K3)$ . Furthermore, since  $\theta_1$  is  $-1$  on  $H^{2,0}(K3)$  and  $H^{0,2}(K3)$  we learn that on  $H^{1,1}(K3)$   $\theta_1$  has eigenvalues  $[(-1)^{10}, (+1)^{10}]$ . Now we can compute  $h^{11}$  of this Calabi-Yau space. We get 10 from the 10  $(1,1)$  forms on  $K3$  with eigenvalue 1 and we get one more from the  $(1,1)$  form on  $T^2$  which is clearly even under the involution. Thus  $h^{1,1} = 11$ . Similarly for  $h^{21}$  we get 10 from taking the wedge product of 10  $(1,1)$  forms with eigenvalue  $-1$  on  $K3$  with the  $(1,0)$  form on  $T^2$  which is odd under  $\theta_2$  and we get one more from taking the wedge product of the  $(2,0)$  form on  $K3$  and the  $(0,1)$  form on  $T^2$ . Thus  $h^{21} = 11$  which of course was required by  $\chi = 0$ . The low-energy theory has gauge group  $U(1)^{12}$  with one of the  $U(1)$  factors being the graviphoton, and  $N_v = 11$  and  $N_h = 12$  (including the dilaton).

The action of the involution  $\theta_1$  on  $K3$  can be usefully summarized as follows. The intersection form on  $K3$  is

$$L_{IJ} = [\Gamma_8 \oplus \Gamma_8 \oplus \sigma^1 \oplus \sigma^1 \oplus \sigma^1]_{IJ}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.2)$$

where  $\Gamma_8$  is the Cartan matrix for  $E_8$ .  $\theta_1$  acts by interchanging two copies of  $\Gamma_8 \oplus \sigma^1$  and as  $-1$  on the third  $\sigma^1$ . This will be useful in the following section when we construct the corresponding orbifold on the heterotic side.

Note that we can choose a point in the moduli space of  $K3$  corresponding to  $T^4/Z_2$  and perform the same  $Z_2$  modding out of  $(T^4/Z_2) \times T^2$  described above. In such a description, the  $Z_2$  acts by reflection on  $T^2$  and its action on  $T^4$  is  $(-1, 1)$  on the complex coordinates  $(z_1, z_2)$  of  $T^4$  as well as a shift by a half lattice vector in both  $z_1$  and  $z_2$  directions. Then the untwisted sector contributes  $(3, 3)$  to  $(h^{11}, h^{21})$  and one of the twisted sectors (corresponding to the first  $Z_2$ ) contributes  $(8, 8)$ .

An important modification of this construction is as follows. The fundamental group of this Calabi-Yau space is  $Z_2$ . This means we can give a Wilson line expectation value to the RR  $U(1)$  field which is present in the ten-dimensional IIA theory. This has no effect whatsoever on string perturbation theory, since all fundamental string states are neutral under this  $U(1)$ . However we will argue later that the theory without the Wilson line is nonperturbatively inconsistent due to problems with “black hole level-matching”. We define  $X$  to be the Calabi-Yau space including the RR Wilson line.

The string tree level moduli space of  $X$  can be computed locally<sup>6</sup>. The moduli space of complex structures is given by the special geometry

$$\mathcal{M}_V = \frac{SU(1, 1)}{U(1)} \times \frac{SO(10, 2)}{SO(10) \times SO(2)}. \quad (2.3)$$

The first (second) factor is the space of complex structures on  $T^2$  (Enriques surface). In the IIA theory these hypermultiplet moduli are augmented by RR scalars which also fill out hypermultiplets. The enlarged hypermultiplet moduli space is determined at tree level from (2.3) and the c-map [25] as the quaternionic space

$$\mathcal{M}_H = \frac{SO(12, 4)}{SO(12) \times SO(4)}. \quad (2.4)$$

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<sup>6</sup> We ignore the issue of global identifications in most of this paper.

The moduli space of complexified Kahler forms on  $X$  is easily computed to leading-order in sigma-model perturbation theory from the cubic intersection form on  $H^2(X)$ . The intersection of the two form on  $T^2$  with the  $\theta_1$  even cohomology of  $K3$  is  $2(\Gamma_8 \oplus \sigma^1)$ . It then follows from the formulae in [26] that the moduli space of vector multiplets is the special geometry  $\mathcal{M}_V$  given in (2.3) above.

Since  $K3$  and  $T^2$  are both their own mirrors, and we are modding out by a free  $Z_2$  action, it is natural to expect that  $X$  is also its own mirror. This is basically true<sup>7</sup> as follows from the analysis of similar orbifolds in [28]. This provides a method to compute the exact Kahler moduli space using the inverse c-map from  $\mathcal{M}_H$  and the fact that  $\mathcal{M}_H$  does not receive world-sheet instanton corrections [29]. This reproduces the leading-order sigma-model result quoted above. We conclude that there can be no worldsheet instanton corrections to the Kahler moduli space, and the exact tree level moduli space is given by  $\mathcal{M}_V \times \mathcal{M}_H$ .

### 3. Soliton Strings and the Duality Dictionary

We now need to know how to map orbifold constructions on the IIA side into orbifold constructions on the heterotic side. String-string duality tells us that the symmetries of the  $K3$  moduli given by  $SO(20, 4; Z)$  are transformed to the same group for the heterotic theory, which implies that we should think of the heterotic lattice as the integral cohomology lattice of  $K3$  and that the Poincare duality on  $K3$  cohomology is what gives the left-right decomposition of the Narain lattice ( $*$  = 1 subspace is right-movers and  $*$  = -1 is the left-movers). Any discrete symmetry of a  $K3$  manifold will be a symmetry of its cohomology lattice and will thus, through this identification, give us the action on the heterotic side, up to potential phases which are represented by ‘shift vectors’ on the heterotic side. This identification of the heterotic lattice with the integral  $K3$  cohomology lattice, as required by string duality, has been found to hold in a physically beautiful way [10,9] realizing the heterotic string as a soliton in the IIA theory.

The soliton string is a fivebrane [18] with four of its spatial extensions wrapping  $K3$ . The worldsheet fields of the soliton string arise as zero modes of the soliton solution. We

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<sup>7</sup> More precisely  $X$  is mirror to itself with a  $Z_2$  discrete torsion turned on, which does not affect the local geometry of moduli space, similar to the situation studied in [27].

will not repeat the entire discussion of [10], but rather mention the relevant points. In the notation of [10] the zero modes are

$$C = \frac{\alpha'}{2\pi} X^I(\sigma) U_I(y) \wedge d e^{2\phi_0 - 2\phi(x)}, \quad (3.1)$$

with  $e^{2\phi(x)}$  the background dilaton field of the string soliton and  $I = 1, \dots, 22$ . The harmonic two-forms  $U_I$  comprise an integral basis for  $H^2(K3, \mathbb{Z})$  with intersection form given by (2.2). The  $X^I$ s are worldsheet fields subject to the chiral constraint

$$\partial_{\pm} X^I = \pm H^I{}_J \partial_{\pm} X^J, \quad (3.2)$$

where  $H$  relates the  $U$  basis to its Hodge dual

$$\hat{*}U_I = U_J H^J{}_I. \quad (3.3)$$

$H$  has signature  $(19, 3)$  (and depends on the modulus of the  $K3$  surface) so there are 19 left-moving and 3 right-moving  $X^I$ s. An additional zero mode involves a combination of  $C$  and the one-form potential  $A$

$$\begin{aligned} A &= \frac{1}{2\pi} X^0(\sigma) d e^{2\phi_0 - 2\phi}, \\ C &= -\frac{1}{\pi} X^0(\sigma) e^{2\phi_0 - 2\phi} H, \end{aligned} \quad (3.4)$$

This gives an additional  $(1, 1)$  (left, right)-moving bosonic zero mode. The gauge transformation law is  $\delta A = d\epsilon$ ,  $\delta C = -2\epsilon H$ , so at zero worldsheet momentum (3.4) corresponds to a gauge transformation with asymptotic parameter  $\frac{X^0}{2\pi}$ . Quantization of RR charge then implies periodicity of  $X^0$ , which we take to be  $X^0 \sim X^0 + 2\pi$ . Finally there is one complex bosonic zero mode  $z^3(\sigma)$  corresponding to transverse motion in the  $T^2$  factor and two more real zero modes (which will not enter in to the discussion) from transverse motion in four-dimensional Minkowski space. Together with the right-moving superpartners this comprises the worldsheet content of a  $T^6$  compactified heterotic string.

Now we translate the action of  $\theta_1$  into an action on the zero mode coordinates of the heterotic string. As discussed below (2.2), the action of  $\theta_1$  exchanges two sets of 10  $U_I$ 's with intersection matrix  $\Gamma_8 \oplus \sigma^1$  and reverses the sign of two more with intersection matrix  $\sigma^1$ . But this is equivalent, from (3.1), to exchanging two sets of 10  $X^I$ 's with momenta living in a lattice  $\Gamma_8 \oplus \sigma^1$  and reversing the sign of two more  $X^I$ 's living on the lattice  $\sigma^1$ . In string theory it is conventional to enlarge the intersection form by working with the full

even-dimensional cohomology, that is by including  $H^0$  and  $H^4$  as well. This enlarges the intersection form (2.2) by an additional  $\sigma^1$  factor. If a RR Wilson line is included,  $\Theta$  is accompanied by a RR U(1) gauge transformation whose square is the identity. The zero momentum part of  $X^0$  is a gauge transformation so inclusion of the Wilson line corresponds to the  $Z_2$  shift  $X^0 \rightarrow X^0 + \pi$  which can be thought of as a shift in the additional  $\sigma^1$  arising from  $H^0$  and  $H^4$ .

#### 4. The $N = 2$ Heterotic String Orbifold

We now wish to describe this as a conventional orbifold of the heterotic string. Working at a general point in the Narain moduli space consistent with the action discussed above we start with an even, self-dual Lorentzian lattice  $\Gamma^{(22,6)}$  of the form

$$\Gamma^{(9,1)} \oplus \Gamma^{(9,1)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(1,1)} \oplus \Gamma^{(2,2)} \quad (4.1)$$

where the two  $\Gamma^{(9,1)}$  factors are isomorphic. We then mod out by a  $Z_2$  action which exchanges the first two factors in (4.1), acts as  $-1$  on the third and fifth factors and as a shift in the fourth factor if we include a RR Wilson line in the type II theory. The  $-1$  action on the fifth factor of (4.1) is inversion of the coordinate  $z^3$  on  $T^2$ . Thus the heterotic orbifold is a  $Z_2$  orbifold of a special Narain compactification which has twelve negative eigenvalues on the left and four on the right.

Now with no shifts the vacuum energy for this twist is  $-1/4$  on the left and zero on the right and thus does not satisfy level-matching. This can be rectified by including a  $Z_2$  shift on the invariant  $\Gamma^{(1,1)}$  of the form  $\delta = (p_L, p_R)/2$  where  $p^2 = p_L^2 - p_R^2 = 2$ . This shifts the left-moving vacuum energy by  $p_L^2/8$  and the right-moving vacuum energy by  $p_R^2/8$ . Since the difference in vacuum energies is now zero level-matching and hence modular invariance are satisfied. We presume that on the IIA side this arises as a non-perturbative consistency condition once an analog of modular invariance is understood for black hole states.

At generic points in the lattice (4.1) the massless spectrum of the orbifold including the shift consists of eleven vector multiplets and twelve hypermultiplets, as predicted by duality. To see this we work in the Ramond-Neveu-Schwarz formalism. Then the right-moving vacuum in the bosonic sector consists of four states with the quantum numbers of the bosonic states of a vector supermultiplet with eigenvalue  $+1$  under the twist and four states with the quantum numbers of the bosonic states of a hypermultiplet with eigenvalue  $-1$  under the twist. Combining these states with the twelve left-moving massless states



with eigenvalues  $-1$  under the twist yields the twelve hypermultiplets. The other states yields the eleven vector multiplets and the gravitational multiplet. The analogous counting goes through for the fermionic states. At generic points there are no massless states in the twisted sector because the right-moving vacuum energy is shifted up by  $p_R^2/8$  which is non-zero at generic points. At tree-level the moduli space is given locally by  $\mathcal{M}_V \times \mathcal{M}_H$ , as follows from the general structure of Narain moduli.

## 5. The Exact Moduli Space and Finite N=2 Theories

We have found that at tree level the moduli space is given by  $\mathcal{M}_V \times \mathcal{M}_H$  in both the heterotic and type II representations. On the heterotic side there are no quantum corrections to hypermultiplets, while on the type II side there are no quantum corrections to vector multiplets. Therefore by duality the moduli space is exact in all expansion parameters.

The absence of worldsheet instanton corrections on the type II side implies the absence of spacetime instanton corrections on the heterotic side<sup>8</sup>. This is possible only if all the Yang-Mills beta functions vanish at all enhanced symmetry points. Thus it must be the case that the states come down to zero mass only in finite representations<sup>9</sup>. Let us check this.

There are two types of enhanced symmetry points which arise at special points in (4.1). The first arises by going to an enhanced symmetry point in  $\Gamma^{(9,1)}$  which in general yields a rank nine simply-laced non-abelian gauge symmetry (e.g.  $E_8 \times SU(2)$ )<sup>10</sup>. Now at such a point it is clear that the symmetric combination of lattice vectors has eigenvalue  $+1$  under the interchange and yields a vector supermultiplet in the adjoint representation. Note that the resulting world-sheet current algebra is now at level two. The antisymmetric combination of lattice vectors also transforms in the adjoint representation of the group

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<sup>8</sup> Other dual pairs of type II heterotic theories constructed by going through enhanced symmetry points will in general have non-trivial instanton corrections. Analysis of these corrections will provide a further stringent consistency check on our proposal.

<sup>9</sup> The gravitational beta functions do not in general vanish. These control higher dimensional  $R^2$  corrections to the leading low-energy effective action of the type studied in [30,31]. Analysis of these effects will lead to yet further checks of our proposed duality.

<sup>10</sup> For example at the moduli of  $X$  corresponding to the orbifold point of  $K3$  mentioned earlier we get an  $SU(2)^8$  non-abelian gauge symmetry.

but since it is odd under the  $Z_2$  it gives a hypermultiplet. This of course is the field content of a finite  $N = 2$  gauge theory.

The other type of enhanced symmetry point yields a more intricate finite theory. We can also go to an enhanced symmetry point in the  $\Gamma^{(1,1)}$  lattice containing the shift  $\delta$  by going to the self-dual radius. At this point lattice vectors are of the form  $(p_L, p_R) = (m + n, m - n)/\sqrt{2}$  with  $m, n$  integers and the shift vector is  $\delta = (1/\sqrt{2}, 0)$  which is half of a root vector of  $SU(2)$ . In the untwisted sector we find a vector supermultiplet in the adjoint representation of  $SU(2)$  from lattice points  $(\pm\sqrt{2}, 0)$  and including the oscillator structure. In the twisted sector we now have massless states from points with  $\tilde{p}_L^2 = 1/2$  and  $\tilde{p}_R^2 = 0$  with  $\tilde{p}_L = (m + n + 1)/\sqrt{2}$  and  $\tilde{p}_R = (m - n)/\sqrt{2}$ . There are two such states with  $m = n = 0$  or  $m = n = -1$  and these two states transform as a doublet of  $SU(2)$ . We next need to compute the multiplicity of these states. There are two contributions coming from the left and right-moving vacuum degeneracies. On the right we now have four anti-periodic and four periodic fermions. Quantization of the zero modes yields a four-fold degenerate ground state and the GSO projection reduces this to two. In addition the  $Z_2$  action on  $\Gamma^{(1,1)} \oplus \Gamma^{(2,2)}$  has eight fixed points which gives a total degeneracy of 16. One can check, using the number of fixed points for asymmetric orbifolds [32], that the interchange of the  $\Gamma^{(9,1)}$  factors does not give any additional degeneracy. This degeneracy is precisely that of four hypermultiplets in the fundamental representation of  $SU(2)$ . As is well known, this is also a finite  $N = 2$  theory which was previously studied in [19]. Some aspects of the duality for this theory found in [19] can also be found in this construction.

One striking aspect of the duality in this context was the observation in [19] that the  $SL(2, Z)$  symmetry of the gauge coupling constant in this example acts also on the flavor symmetry which in this example is  $Spin(8)$ . However, a subgroup of  $SL(2, Z)$ ,  $\Gamma(2)$ , acts only on the coupling constant and does not act on the flavors. Moreover the extra elements of  $SL(2, Z)$  given by reduction mod 2 form the permutation group  $S_3$  on three objects, which acts as triality on the  $Spin(8)$  representations. Let us see whether we can see any hints of this structure in our model.

First we note that  $\Gamma(2)$  is the subgroup of  $SL(2, Z)$  preserving the spin structures on the torus and the four conjugacy classes of  $Spin(8)$  can be identified with the four fixed points of a  $T^2$  under the involution with  $S_3$  permuting three of the fixed points (it does not act on the one at the origin).

According to the dictionary of string-string duality applied to compactifications on a  $T^2$  down to 4 dimensions [6,4] the dilaton of the heterotic theory gets mapped to the

Kahler class of  $T^2$  on the type II side. Here we have divided further by a  $Z_2$  acting as reflection on  $T^2$  and an involution on  $K3$ . Since the  $Z_2$  involution on  $T^2$  exists for all complex and Kahler structures, this in particular means that the Kahler class of  $T^2$  survives the modding out, and it thus is still parametrized by the fundamental domain of the torus modulus. This means that the heterotic string coupling constant should likewise still be parametrized by the fundamental domain of the upper-half plane. However, we are asking for more information. In particular  $SL(2, Z)$  transformations on the coupling constant should also act as triality as described with only the  $\Gamma(2)$  subgroup leaving the flavors untouched. Note that, due to mirror symmetry on  $X$  this is equivalent to the same question about the  $SL(2, Z)$  which acts on the complex structure of  $T^2$  which is more easily realized geometrically. Note that the 4 hypermultiplets of  $SU(2)$  on the heterotic side came from the four fixed points of  $T^2$ . Given that  $T^2$  is common to both type II and heterotic part, it thus follows that the subgroup of  $SL(2, Z)$  that does not act on the flavors, i.e. does not reshuffle the fixed points, is just  $\Gamma(2)$ , as expected from [19]. In order to verify that the extra elements of  $SL(2, Z)$  do act as triality on the flavor representations, we have to study more carefully how the  $Spin(8)$  is realized on the fixed points of the asymmetric orbifold of the heterotic theory. In particular physical vertex operators are particular linear combinations of fixed point sets [32]. A careful study following the above strategy should lead to the triality action of [19]. As a further check on these ideas, note that at the other type of enhanced gauge symmetry points (e.g.  $E_8 \times SU(2)$ ) the matter does not come from the fixed point sets and thus there is no action of  $SL(2, Z)$  on the flavor symmetry, as expected from  $N = 4$  duality.

Finally, we note that the  $N_f = 4$ ,  $SU(2)$  theory also has a Higgs branch along which the  $SU(2)$  doublets condense and break the  $SU(2)$  gauge symmetry. Since we start out with 4 doublets of  $SU(2)$ , this leaves us after breaking  $SU(2)$  with  $8 - 3 = 5$  extra hypermultiplets. Moreover we have gotten rid of one vector multiplet (corresponding to the  $U(1)$  of  $SU(2)$ ). In the Type IIA theory this should correspond to black holes becoming massless and condensing. Assuming this new phase corresponds to a Calabi-Yau compactification, it would have to be one with Hodge numbers  $(h^{11}, h^{12}) = (10, 16)$ . This Calabi-Yau space is not self-mirror and should have world-sheet instanton corrections on the Type IIA side and spacetime quantum corrections on the heterotic side which are related by second-quantized mirror symmetry.

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